Problem 1. Let $r_{101}(x)$ be the remainder x leaves upon division by 101. For example, $r_{101}(2022) = 2$. Let $a_1, a_2, \ldots, a_{50}, b_1, b_2, \ldots b_{50}$ be pairwise distinct positive integers less than or equal to 100. Compute the smallest possible value of

$$r_{101}(a_1b_1) + r_{101}(a_2b_2) + \cdots + r_{101}(a_{50}b_{50}).$$

Problem 2. Let T be the number you will receive. Compute the number of monic quartic polynomials $P(x)$ with integer coefficients such that $P(x)$ divides $x^T - 1$.

Problem 3. Let T be the number you will receive. Let $ABCDEF$ be a regular hexagon, and let P be an interior point. Suppose that the distance from P to line AB is 5, the distance from P to line BC is 8, and the distance from P to line CD is T. Compute the area of the circumcircle of $ABCDEF$.

Problem 4. Let T be the number you will receive. It is your turn in a game of Nim. There are four piles. The first pile has 91 tokens, the second pile has $\frac{T}{\pi}$ tokens, the third pile has 119 tokens, and the fourth pile has 3 tokens. If you want to guarantee that you will win, then there are three possible moves you could make: you could remove a tokens from the first pile, or b tokens from the second pile, or c tokens from the third pile. Compute the ordered triple (a, b, c).