Hop on MOP

1. Fox is putting on his six socks and shoes. He has six legs, and wishes to put a sock and a shoe on each leg, so that each leg's sock comes before its shoe. He must obey the following condition: it is not permitted for there to be legs A and B such that Fox puts a sock on leg A, then later puts a sock on leg B, then later a shoe on leg B, then later a shoe on leg A. Find the number of ways for Fox to put on his socks and shoes.

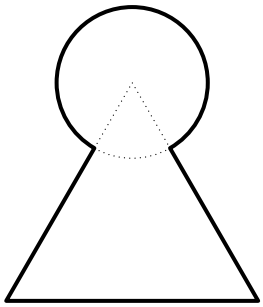
2. Sam-I-Am is trying to serve you eggs. He asks, "Would you eat them in a box? A cubical box of side length 1 foot?"

   If he picks a point P uniformly and randomly in the interior of this box, let $\frac{m}{n}$ the probability that a spherical egg of radius 2 inches with center P would fit entirely inside the box, where m and n are relatively prime positive integers. Find $m+n$.

3. Diver Getz and Diver Gitz give you a Time-Telling Fish. Its face is an analog clock. Right now, the angle between the clock's hands is d degrees, where d is a positive integer that is divisible by 7. Also, $2d$ minutes ago, the time was exactly $6:00$. Find the largest possible value of d.

4. Mayzie is currently 5 miles east of her nest, and 15 miles west of Florida. She is indecisive, so every hour, she flies 1 mile east or west, with equal probability. If she reaches her nest then she hatches her egg; if she reaches Florida then she forgets about her egg and goes on vacation (in either case, she stops flying). Find the expected distance that Mayzie flies.

5. The Key-Slapping Slippard lives in a keyhole, which is the union of a circle of radius 3, and an equilateral triangle of side length 10, one of whose vertices is the center of the circle:

   If the Key-Slapping Slippard is shaped as a rectangle with a horizontal base, then its maximum possible area can be written as $\frac{m\sqrt{n}}{p}$, where m,

n, and p are positive integers, m and p are relatively prime, and n is squarefree. Find $m+n+p$.

6. Mr. Brown can moo, and he also has a function $f : \{1,2,3,4,5,6\} \to \{1,2,3,4,5,6\}$ such that

$$f(f(f(f(f(x))))) = x$$

for each $x \in \{1,2,3,4,5,6\}$. Find the number of possible functions Mr. Brown has.

7. You are at $(0,0)$ on the coordinate plane, and are attempting to reach the Boom Bands at $(1,0)$. However, you are lost, so for every step, you choose a direction (left, right, up, or down) uniformly at random, and move one unit in that direction. The points with $y < 0$ form the Waiting Place; if you reach such a point, you stop moving. If the probability that you are at the Boom Bands after exactly 3 steps is $\frac{m}{n}$, where m and n are relatively prime positive integers, find $m+n$.

8. Kitty O'Sullivan Krauss wishes to make her swimming pool in the shape of a regular n-gon, where n is a positive three-digit integer. Given that a regular n-gon is constructible using ruler and compass, find the minimum possible value of n.

9. The Yink likes to drink pink ink. Pink ink is made by mixing red ink and white ink in a certain fixed proportion. The amount of white ink in 96 ounces of pink ink is equal to the amount of red ink that would be mixed with 100 ounces of white ink to make some amount of pink ink. Find the number of ounces of white ink in 40 ounces of pink ink.

10. The Lorax speaks for the trees! In particular, the Lorax speaks for the unlabeled rooted trees with eight vertices. Find the number of trees for which the Lorax speaks.

11. Initially, stars are always in style. However, Sylvester McMonkey McBean decides to change this, in order to generate more business. In a 365-day year, he first declares that every day is a Star-Off day, when stars are out of style. He then declares that every second day is a Star-On day (when stars are in style). He then toggles the status of every third day, every fourth day, and so on, up to every 365th day. After he is finished with this process, find the number of Star-Off days in the year.

12. There's a Wocket in my Pocket, and it likes to factor polynomials. Find the eighteenth-smallest positive integer n such that the polynomial $x^4 - nx^2 + 1$ can be factored into two nonconstant polynomials with integer coefficients.

13. Thidwick lives in a herd of n moose (including himself), with $n \geq 60$. After 11 animals start living on his antlers, he notices that the total number of animals is a square, and the total number of antlers is eight less than a

square. (Assume that every moose has two antlers, and none of the 11 other animals have antlers.) Find the smallest possible value of n.

14. Yertle is at the top of a stack of turtles, with Mack at the bottom; Yertle is unhappy to notice a tree taller than his stack. "My distance to the top of the tree is a hundred more than a third of your distance to the base of the tree," says Mack to Yertle. "That's interesting, because the height of the tree is a hundred more than my height, as well!" says Yertle. "And in fact, my distance to the base of the tree is an integer," says Mack.

    Find the maximum possible distance from Yertle to the base of the tree.

15. Tired of waiting for Miss Becker, you realize that both your age and Miss Becker's age are prime, and her current age is exactly half of what your age was one year ago. Furthermore, all one-digit primes are a (nonzero) quadratic residue modulo exactly one of your age and her age. Given this information, find your minimal possible age.